Power-counting and the validity of the classical approximation during inflation

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Abstract: We use the power-counting formalism of effective field theory to study the size of loop corrections in theories of slow-roll inflation, with the aim of more precisely identifying the limits of validity of the usual classical inflationary treatments. We keep our analysis as general as possible in order to systematically identify the most important corrections to the classical inflaton dynamics. Although most slow-roll models lie within the semiclassical domain, we find the consistency of the Higgs-Inflaton scenario to be more delicate due to the proximity between the Hubble scale during inflation and the upper bound allowed by unitarity on the new-physics scale associated with the breakdown of the semiclassical approximation within the effective theory. Similar remarks apply to curvature-squared inflationary models.

Contents

1 Introduction

The hypothesis that the universe underwent accelerated expansion during an early inflationary epoch can explain the flatness, isotropy, homogeneity, horizon and undesired relic problems of the early universe. Typically, inflation is caused by a local Lorentz invariant energy density dominating the equation of state and driving an exponential expansion of the comoving Hubble length [1]. Even better, the growth of quantum fluctuations during inflation allows a simple description of the observed features of the primordial cosmological fluctuations that are required in the Hot Big Bang to seed the large-scale structure observed in the universe. The general predictions of inflationary scenarios also agree with the increasingly precise observations of the properties of the Cosmic Microwave Background (CMB), such as measured most recently by WMAP [2]. While the idea of inflation is in good qualitative and quantitative agreement with the data, it has so far proven more difficult to embed inflation within a more complete framework of physics at the very high energies that are required.

Thus, many inflationary scenarios exist that are constructed to be consistent with the current experimental constraints. The vast majority of these fall into the category of 'slow-roll' inflation, for which a scalar field (inflaton), classically evolves under the influence of a very flat potential. It is the approximately constant energy density of the scalar during this classical slow roll that drives the inflationary epoch. Although some scenarios are more sophisticated, and have incorporated important quantum effects modifying or generating the potential, the usage of the semi-classical approximation is standard in inflation studies.

In this paper, we apply the power counting formalism of effective field theory to study the question of the size of the loop corrections of the scalars coupled to gravity that are commonly employed in the inflation literature. Our results are constructed to be as general as possible, are not limited to one loop, and allow one to directly examine the quantum corrections of physical quantities (like the classical inflaton potential or scattering cross sections). The techniques used rely on simple dimensional arguments that are known to work for similar applications of non-renormalizable theories in non-gravitational situations (like chiral perturbation theory in the strong interactions). The beauty of the approach is its simplicity, since the constraints on couplings and masses that underlie the validity of the semi-classical approximation can be quickly determined using power-counting arguments without the need for extensive explicit calculation (and yet agrees with these calculations when they are available).

As an example of the utility of the formalism we develop, we study the unusually predictive and simple Higgs-Inflaton scenario [3]. In this scenario, it is the Standard Model's Higgs boson itself that acts as the inflaton, a scenario that is made possible through the addition of the single dimension-four interaction, $\delta\mathcal{L} = \xi H^\dagger H\, R$, that is usually neglected, but that is expected to be *required* to exist due to renormalization of the theory in curved space [4]. This term encodes the experimentally untested possibility of a large nonminimal coupling of the Higgs to gravity, and the freedom to choose the new coupling ξ is all that is required to ensure an inflationary slow roll.

However, these conclusions are drawn using a semi-classical analysis, and we show that the domain of validity of this approximation is very narrow for this model due to the large size of $\xi \simeq 10^4$ required for successful inflation (consistent with WMAP constraints). We find that the semiclassical analysis requires that the scale M, defining the limit of validity of the effective theory, lies in the narrow window $M_p/\xi \gg M \gg \sqrt{\lambda_H} M_p/\xi$, where λ_H is the usual quartic self-coupling of the Higgs in the Standard Model potential. We show how this condition is very sensitive to the existence of other heavy particles in the microscopic theory that couple to the Higgs, even if these couplings are quite weak. Similar remarks apply to curvature-squared inflationary models, which also walk a thin line of consistency.

2 Power-counting

Power counting the scales that appear in loops is a standard technique of effective field theory, for which many excellent reviews exist [5] in the literature, including applications to gravity [6, 7]. In this section, we use power counting to identify how successive terms in the semiclassical expansion depend on the various scales and couplings of the inflationary theory of interest. There are two types of effective field theories normally considered in the literature for inflation, that differ according to whether or not they focus on the complete inflaton-metric system [8], or on the specific adiabatic mode which (for single-field models) controls the spectrum of primordial perturbations [9]. We here consider theories of the form of [8] (and its multi-scalar generalizations), and provide a power-counting analysis of the order in the low-energy expansion at which any effective interaction contributes.

2.1 The effective field theory

For definiteness, consider the following effective lagrangian, describing the low-energy interactions of N dimensionless scalar fields, θ^i, and the metric, $g_{\mu\nu}$:

$$-\frac{\mathcal{L}_{\text{eff}}}{\sqrt{-g}} = v^4 V(\theta) + \frac{M_p^2}{2} g^{\mu\nu} \Big[W(\theta) R_{\mu\nu} + G_{ij}(\theta) \partial_\mu \theta^i \partial_\nu \theta^j \Big] \tag{2.1}$$

$$+ A(\theta)(\partial\theta)^4 + B(\theta) R^2 + C(\theta) R (\partial\theta)^2 + \frac{E(\theta)}{M^2} (\partial\theta)^6 + \frac{F(\theta)}{M^2} R^3 + \cdots .$$

Here the lagrangian is organized as a derivative expansion, with terms involving up to two derivatives written explicitly and the rest only written schematically in order to sketch the dimension of the coefficients. In particular R^3 collectively represents all possible independent invariants constructed from three Riemann tensors, or two Riemann tensors and two of its covariant derivatives; $R(\partial\theta)^2$ denotes all possible invariants involving one power of the Riemann tensor and two derivatives acting on θ^i; and so on for the other terms.

In eq. (2.1) the scalar fields are normalized so that the coefficient of their kinetic terms is the reduced Planck mass, defined in terms of Newton's constant by[1] $M_p = (8\pi G)^{-1/2}$. All of the coefficient functions, $V(\theta)$, $G_{ij}(\theta)$, $A(\theta)$ and so on, are dimensionless, and the scale M that makes up the dimensions is taken to be characteristic of whatever underlying microscopic physics has been integrated out.[2] Since it is the *smallest* mass that dominates in such a denominator, it is important to recognize that generically $M \ll M_p$ [6]. In applications to inflation our interest is usually (but not always) in situations where $V \simeq v^4 \ll M^4$ when $\theta \simeq \mathcal{O}(1)$.

For the purposes of estimating the size of quantum effects, we expand about a classical solution,

$$\theta^i(x) = \vartheta^i(x) + \frac{\phi^i(x)}{M_p} \quad \text{and} \quad g_{\mu\nu}(x) = \hat{g}_{\mu\nu}(x) + \frac{h_{\mu\nu}(x)}{M_p}, \tag{2.2}$$

which allows the effective action, eq. (2.1), to be written as a sum of effective interactions

$$\mathcal{L}_{\text{eff}} = \hat{\mathcal{L}}_{\text{eff}} + M^2 M_p^2 \sum_n \frac{c_n}{M^{d_n}} \mathcal{O}_n \left(\frac{\phi}{M_p}, \frac{h_{\mu\nu}}{M_p} \right) \tag{2.3}$$

where $\hat{\mathcal{L}}_{\text{eff}} = \mathcal{L}_{\text{eff}}(\vartheta, \hat{g}_{\mu\nu})$ is the lagrangian density evaluated at the background configuration. The sum over n runs over the labels for a complete set of interactions, $\mathcal{O}_n$, each of which involves $N_n = N_n^{(\phi)} + N_n^{(h)} \geq 2$ powers of the fields ϕ^i and $h_{\mu\nu}$. ($N_n \neq 1$ follows as a consequence of the background field equations for ϑ^i and $\hat{g}_{\mu\nu}$.) The parameter d_n counts the number of derivatives appearing in $\mathcal{O}_n$, and so the factor M^{-d_n} is what is required to keep the coefficients, c_n, dimensionless. The overall prefactor, $M^2 M_p^2$, is chosen so that the kinetic terms — i.e. those terms in the sum for which $d_n = N_n = 2$ — are M and M_p independent. Notice also that the operators $\mathcal{O}_n$ depend implicitly on the properties of the classical backgrounds, ϑ^i and $\hat{g}_{\mu\nu}$, about which the expansion is performed.

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[1]This normalization is convenient for large-field inflationary models, for which the scalars move over Planckian distances in field space, but we also consider scalars whose couplings are stronger than Planck-suppressed in what follows below by including couplings that carry compensating powers of M_p.

[2]That is, M might be regarded as the lightest of the particles that were integrated out to produce the low-energy theory. Our calculations below show why such a mass would appear in this way.

Comparing eqs. (2.1) and (2.3) also shows that there are factors of the scales v, M and M_p buried in the dimensionless coefficients c_n. In particular, any term involving no derivatives comes from the scalar potential, $V(\theta)$, and so

$$c_n = \left(\frac{v^4}{M^2 M_p^2}\right) \lambda_n \qquad (\text{if } d_n = 0)\,, \tag{2.4}$$

where the λ_n represent dimensionless couplings that are independent of M_p and M. Similarly, the absence of M_p in all of the terms involving more than two derivatives in eq. (2.1) implies

$$c_n = \left(\frac{M^2}{M_p^2}\right) g_n \qquad (\text{if } d_n > 2)\,, \tag{2.5}$$

where g_n is similarly independent of M and M_p.

In terms of the λ_n's the scalar potential has the schematic form

$$V(\phi) = v^4 \left[\lambda_0 + \lambda_2 \left(\frac{\phi}{M_p}\right)^2 + \lambda_4 \left(\frac{\phi}{M_p}\right)^4 + \cdots\right]\,, \tag{2.6}$$

which shows that the natural scale for the scalar masses under the above assumptions is $m \simeq v^2/M_p$. The quartic coupling constant, $\lambda_4 (v/M_p)^4$, is similarly Planck suppressed. Such small masses and couplings follow from the assumption that V only runs through a range of order v^4 as ϕ runs all the way out to M_p. Although such a shallow potential often arises in inflationary applications, in some circumstances it is also interesting to consider potentials for which $V \sim v^4$ when ϕ runs over a comparable range, $\phi \sim v$, and so for which $m \simeq v$ up to dimensionless couplings. Such potentials can be included in the above analysis by further redefining

$$\lambda_n = \left(\frac{M_p}{v}\right)^{\check{N}_n} \check{\lambda}_n\,, \tag{2.7}$$

in the power-counting rules that are to follow. Here $\check{N}_n \leq N_n$ denotes the number of scalar fields of this type appearing in the vertex in question. This need not agree with N_n if there are also other scalars, or graviton vertices, appearing in the $d_n = 0$ vertex of interest.

2.2 Semiclassical perturbation theory

Our goal is to follow how the couplings c_n and the scales M and M_p appear in physical quantities at various orders of the semiclassical expansion. To this end we divide $\mathcal{L}_{\text{eff}}$ into an unperturbed and perturbed lagrangian density,

$$\mathcal{L}_{\text{eff}} = \left(\hat{\mathcal{L}}_{\text{eff}} + \mathcal{L}_0\right) + \mathcal{L}_{\text{int}}\,, \tag{2.8}$$

where $\mathcal{L}_0$ consists of those terms in $\mathcal{L}_{\text{eff}}$ for which $N_n = 2$ and $d_n \leq 2$. Since the path integral over ϕ^i and $h_{\mu\nu}$ is Gaussian in the absence of $\mathcal{L}_{\text{int}}$ we can define the semiclassical expansion in principle by computing the generator, Γ, of 1-particle irreducible (1PI) graphs perturbatively in $\mathcal{L}_{\text{int}}$. This is a semiclassical expansion because the leading contribution is the classical result

$$\Gamma[\theta, g_{\mu\nu}] = \int \mathrm{d}^4 x \, \mathcal{L}_{\text{eff}}(\theta, g_{\mu\nu}) + \cdots\,. \tag{2.9}$$

The key issue is to identify what the small quantity is that makes such an expansion a good approximation. To determine this, imagine now computing a contribution to Γ coming from a Feynman graph involving $\mathcal{E}$ external lines. The propagators, $G(x, y)$, associated with each of the I internal lines in this graph come from inverting the differential operator that is defined by the term $\mathcal{L}_0$. The important thing about these for the present purposes is that they do not depend on M and M_p, although they can depend on scales (like the Hubble scale, H) that appear in the background configurations, ϑ^i and $\hat{g}_{\mu\nu}$.

Similarly, vertices in this graph all come from terms in $\mathcal{L}_{\rm int}$, and so each time the interaction $\mathcal{O}_n$ contributes a vertex to the graph it comes with a factor of $c_n M_p^{2-N_n} M^{2-d_n}$. If the graph contains a total of V_n such vertices it acquires in this way a factor

$$\prod_n \left[c_n M_p^{2-N_n} M^{2-d_n} \right]^{V_n} = M_p^{2-2L-\mathcal{E}} \prod_n \left[c_n M^{2-d_n} \right]^{V_n} , \qquad (2.10)$$

where the equality uses the identity

$$2I + \mathcal{E} = \sum_n N_n V_n \qquad (2.11)$$

that expresses that the end of each line in the graph must occur at a vertex, as well as the definition,

$$L = 1 + I - \sum_n V_n , \qquad (2.12)$$

of the number of loops, L, of the graph.

Power-counting. The relative contribution of each graph to Γ is then simplest to enumerate using dimensional arguments. However any such argument is complicated by the ultraviolet divergences that arise in the integration over the positions, x, of the vertices; divergences that may be traced to the singularities in the propagators, $G(x, y)$, in the coincidence limit $y \to x$. For the purposes of making the dimensional argument it is therefore simplest to regularize these divergences using dimensional regularization, since in this case all of the dimensions of the various integrations is set by a physical scale appearing in the problem (such as the masses of the fields ϕ^i, or a scale like H characterizing the size of a derivative of the background classical configuration).[3]

Suppose now that E denotes the largest of the physical scales that appear explicitly in the propagators or vertices of the calculation. Then to leading approximation we can neglect any other, smaller, scales compared with E when estimating the size of a particular Feynman graph. Since the contributions to Γ all share the same dimension as the initial

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[3]Naively, using a cutoff to regulate these divergences would seem to change the estimates we are about to make. However the cutoff-dependent estimates found in this way are guaranteed to cancel cutoff-dependent counter-terms once the theory is renormalized, since physical quantities cannot depend on how we choose to arbitrarily regulate a graph. What counts physically is how observables depend on observable (or renormalized) quantities, and using a cutoff regularization simply makes it difficult to follow dimensional analysis through intermediate steps of the calculation. Of course the final answer does not depend on how the calculation is performed, and any strong dependence on a cutoff in the regularized theory shows up in dimensional regularization as a dependence on a large physical scale in the problem, such as the mass of a heavy particle that has been integrated out.

lagrangian density $\mathcal{L}_{\text{eff}}$, the contribution of a graph involving $\mathcal{E}$ external lines, L loops and V_n vertices involving d_n derivatives becomes[4]

$$\mathcal{A}_{\mathcal{E}}(E) \simeq E^2 M_p^2 \left(\frac{1}{M_p}\right)^{\mathcal{E}} \left(\frac{E}{4\pi M_p}\right)^{2L} \prod_n \left[c_n \left(\frac{E}{M}\right)^{d_n-2}\right]^{V_n}. \tag{2.13}$$

The factors of 4π in this expression come from standard arguments. (For example, for a flat background with constant ϑ^i, they arise from the loop-integral measure in momentum space, $\int \mathrm{d}^4p/(2\pi)^4$, once the angular integration over the momentum direction is taken into account.)

Keeping in mind the factors of M and M_p that are hidden in some of the c_n's — c.f. eqs. (2.4) and (2.5) — it is useful to write separately the terms with $d_n = 0$ and $d_n = 2$ in the product, to get

$$\mathcal{A}_{\mathcal{E}}(E) \simeq E^2 M_p^2 \left(\frac{1}{M_p}\right)^{\mathcal{E}} \left(\frac{E}{4\pi M_p}\right)^{2L} \prod_{d_n=2} \left(c_n\right)^{V_n} \tag{2.14}$$

$$\times \prod_{d_n=0} \left[\lambda_n \left(\frac{v^4}{E^2 M_p^2}\right)\right]^{V_n} \prod_{d_n\geq 4} \left[g_n \left(\frac{E}{M_p}\right)^2 \left(\frac{E}{M}\right)^{d_n-4}\right]^{V_n}.$$

Eq. (2.14) is the main result of this section. It shows in particular what combination of scales must be small in order to justify the validity of the perturbative expansion. A generic sufficient condition for successive insertions of interactions to be smaller than preceding ones is to have E be sufficiently small,

$$\frac{E}{4\pi M_p} \ll 1, \tag{2.15}$$

and

$$g_n \left(\frac{E}{M_p}\right)^2 \left(\frac{E}{M}\right)^{d_n-4} \ll 1 \quad (\text{for } d_n \geq 4). \tag{2.16}$$

Repeated insertions of two-derivative interactions do not generically generate large contributions provided

$$c_n \ll 1 \quad (\text{for } d_n = 2), \tag{2.17}$$

although having $c_n \simeq \mathcal{O}(1)$ need not cause problems if symmetries strongly constrain the kinds of interactions of this kind that can arise. For example, for pure gravity only the Einstein-Hilbert action itself has two derivatives, for which all the resulting graviton interactions have c_n's of order one. The lack of suppression of these interactions shows that they are all generically equally important in a given low-energy process.[5]

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[4]For simple backgrounds these calculations can be made in momentum space (although the dimensional argument being made does not require this), and when this is done the reader should note that eq. (2.13) pulls out the standard overall momentum-conserving factor, $(2\pi)^4 \delta^4(q)$, from $\mathcal{A}_{\mathcal{E}}(E)$, where q denotes the total 4-momentum flowing into the graph.

[5]That is, although the low-energy expansion controls higher derivatives, for generic relativistic applications in General Relativity one must work to all orders in the expansion of the metric about a given background, $g_{\mu\nu} = \overline{g}_{\mu\nu} + h_{\mu\nu}$.

Finally, the only place where inverse powers of E arise is associated with no-derivative interactions, and *a-priori* these seem like they could be dangerous in a low-energy expansion since

$$\lambda_n \left(\frac{v^4}{E^2 M_p^2} \right) \ll 1 \quad \text{(for } d_n = 0\text{)} \tag{2.18}$$

might not be satisfied. This would be even more worrisome in the event that the potential has the form $V = v^4 f(\phi/v)$ for some order-one function $f(x)$, since in this case we have seen — c.f. eq. (2.7) — that we must take $\lambda_n = (M_p/v)^{\check{N}_n} \check{\lambda}_n$ for vertices involving these scalars. Although it is true that low-energy is not itself sufficient to suppress these interactions, their presence need not destroy the low-energy approximation due to correlations that the topology of a graph imposes amongst the numbers of loops, the number of vertices and the number of external lines, as we now see.

For example, imagine the potential worst-case scenario for the low-energy expansion where all of the vertices of the Feynman graph have $d_n = 0$ and $\check{N}_n = N_n$ (i.e. only involve the largest and most dangerous couplings). In this case the identities (2.11) and (2.12) hold separately for the internal lines and vertices involving only the dangerous scalar, and so

$$\sum_n (\check{N}_n - 2)\check{V}_n = \check{\mathcal{E}} - 2 + 2L \,, \tag{2.19}$$

leading to

$$\begin{aligned}
\mathcal{A}_{\check{\mathcal{E}}}(E) &\simeq E^2 M_p^2 \left(\frac{1}{M_p} \right)^{\check{\mathcal{E}}} \left(\frac{E}{4\pi \, M_p} \right)^{2L} \prod_{d_n=0} \left[\check{\lambda}_n \left(\frac{M_p}{v} \right)^{\check{N}_n} \left(\frac{v^4}{E^2 M_p^2} \right) \right]^{\check{V}_n} \\
&\simeq E^2 v^2 \left(\frac{1}{v} \right)^{\check{\mathcal{E}}} \left(\frac{E}{4\pi \, v} \right)^{2L} \prod_{d_n=0} \left[\check{\lambda}_n \left(\frac{v^2}{E^2} \right) \right]^{\check{V}_n} .
\end{aligned} \tag{2.20}$$

Clearly all powers of M_p have dropped out in this expression and, as we see in more detail below, provided $\phi \simeq E$ the net power of E/v that appears in Γ is then

$$\left(\frac{E}{v} \right)^{2+\check{\mathcal{E}}+2L-\sum_n 2\check{V}_n} = \left(\frac{E}{v} \right)^{4+\sum_n (\check{N}_n-4)\check{V}_n} \,, \tag{2.21}$$

which uses eq. (2.19) once more. This shows that quintic and higher interactions generate only positive powers of E/v, while quartic interactions are neither enhanced nor suppressed by E/v (and so must be controlled purely by the small size of the relevant dimensionless couplings, $\check{\lambda}_n$).

Since there are no interactions with $N_n = 1$ (by virtue of the background field equations) or $N_n = 2$ (as these are 'mass' terms in the unperturbed lagrangian density), only the super-renormalizable cubic terms with $N_n = 3$ are potentially dangerous to the low-energy expansion (unless their dimensionless coefficients are also suppressed so that $\check{\lambda}_3 \simeq \mathcal{O}(E/v)$). Such trilinear vertices can indeed cause trouble for the low-energy expansion, if they are of order $\lambda_3 v^4 (\phi/M_p)^3 \simeq \check{\lambda}_3 v \phi^3$, since v need not be small compared with the low-energy scales, E, to which the effective theory is applied.

2.3 Examples

Eq. (2.14) has a number of interesting special cases.

Pure gravity with no cosmological constant. The only thing in the above arguments to change in the case of pure gravity (i.e. no scalar fields) in the absence of a cosmological constant is the absence of interactions having $d_n = 0$. In this case eq. (2.14) reproduces the standard result for General Relativity [6]. It predicts, in particular, that for any $\mathcal{E}$ the dominant contributions arise for $L = 0$ with only vertices satisfying $d_n = 2$ included. For pure gravity these graphs amount to working with General Relativity in the purely classical limit. The first sub-leading contributions may be similarly found, and correspond to working with General Relativity at one loop (i.e. with $L = 1$ and $V_n = 0$ unless $d_n = 2$), or working at classical level and allowing precisely one insertion from a curvature-squared interaction (i.e. with $L = 0$ and $V_n = 0$ for $d_n > 4$, $V_n = 1$ for $d_n = 4$ and V_n arbitrary if $d_n = 2$).

Integrating out a particle of mass $m \ll M$. Another application specializes to the case where the largest scale in the amplitude is the mass, m, of a particle that is being integrated out. In this case provided all other scales are much smaller than m the result for the Γ is local, and expression (2.13) or (2.14) can be regarded as describing how effective interactions are renormalized in $\mathcal{L}_{\text{eff}}$ due to the removal of this particle. More quantitatively, if

$$V(\theta) = v^4 \left[\lambda_0 + \lambda_2 \theta^2 + \lambda_4 \theta^4 + \cdots \right] = v^4 \lambda_0 + \frac{\lambda_2 v^4}{M_p^2} \phi^2 + \frac{\lambda_4 v^4}{M_p^4} \phi^4 + \cdots , \qquad (2.22)$$

with all λ_n's being of order unity, then the masses of the θ^i particles are of order $m \simeq v^2/M_p$. To make one particle systematically heavy relative to the others, we either require $\lambda_2 \gg 1$ for the heavy field (as above, where $\lambda_2 = (M_p/v)^2 \check{\lambda}_2$, say) or $\lambda_2 \ll 1$ for all of the others.

Since the largest scale in the Feynman graphs is m by assumption, we may use the above power-counting estimates with $E \simeq m$. Furthermore, if we focus on contributions to $\mathcal{A}_{\mathcal{E}}$ that involve precisely D derivatives, denoted $\mathcal{A}_{\mathcal{E}}^D$, then the same dimensional arguments as above predict the following scaling:

$$\mathcal{A}_{\mathcal{E}}^D \simeq m^2 M_p^2 \left(\frac{\partial}{m} \right)^D \left(\frac{1}{M_p} \right)^{\mathcal{E}} \left(\frac{m}{4\pi M_p} \right)^{2L} \prod_n \left[c_n \left(\frac{m}{M} \right)^{d_n - 2} \right]^{V_n}$$

$$\simeq m^2 M_p^2 \left(\frac{\partial}{m} \right)^D \left(\frac{1}{M_p} \right)^{\mathcal{E}} \left(\frac{m}{4\pi M_p} \right)^{2L} \prod_{d_n = 2} \left(c_n \right)^{V_n} \qquad (2.23)$$

$$\times \prod_{d_n = 0} \left[\lambda_n \left(\frac{v^4}{m^2 M_p^2} \right) \right]^{V_n} \prod_{d_n \geq 4} \left[g_n \left(\frac{m}{M_p} \right)^2 \left(\frac{m}{M} \right)^{d_n - 4} \right]^{V_n} .$$

Comparing this with the coefficients of the effective interaction valid below the scale m, defined using the form of eq. (2.3) (but with M replaced by m)

$$\tilde{\mathcal{L}}_{\text{eff}} = \hat{\mathcal{L}}_{\text{eff}} + m^2 M_p^2 \sum_n \frac{\tilde{c}_n}{m^{\tilde{d}_n}} \mathcal{O}_n \left(\frac{\phi}{M_p}, \frac{h_{\mu\nu}}{M_p} \right) , \qquad (2.24)$$

we see the Feynman graph in question contributes

$$\delta \tilde{c}_n \simeq \left(\frac{m}{4\pi \, M_p} \right)^{2L} \prod_n \left[c_n \left(\frac{m}{M} \right)^{d_n - 2} \right]^{V_n} . \tag{2.25}$$

In terms of the m- and M_p-independent, dimensionless couplings, $\tilde{\lambda}_n$, $\tilde{g}_n$, λ_n and g_n, these become

$$\delta \tilde{\lambda}_n \simeq \left(\frac{m^2 M_p^2}{v^4} \right) \left(\frac{m}{4\pi \, M_p} \right)^{2L} \prod_{d_n = 2} \left(c_n \right)^{V_n} \prod_{d_n = 0} \left[\lambda_n \left(\frac{v^4}{m^2 M_p^2} \right) \right]^{V_n} \prod_{d_n \geq 4} \left[g_n \left(\frac{m}{M_p} \right)^2 \left(\frac{m}{M} \right)^{d_n - 4} \right]^{V_n} \tag{2.26}$$

while for $\tilde{d}_n \geq 4$ we instead have

$$\delta \tilde{g}_n \simeq \left(\frac{M_p^2}{m^2} \right) \left(\frac{m}{4\pi \, M_p} \right)^{2L} \prod_{d_n = 2} \left(c_n \right)^{V_n} \prod_{d_n = 0} \left[\lambda_n \left(\frac{v^4}{m^2 M_p^2} \right) \right]^{V_n} \prod_{d_n \geq 4} \left[g_n \left(\frac{m}{M_p} \right)^2 \left(\frac{m}{M} \right)^{d_n - 4} \right]^{V_n} . \tag{2.27}$$

For example, at tree level ($L = 0$) the corrections to couplings in the scalar potential ($\tilde{d}_n = 0$) are of order

$$\delta \tilde{\lambda}_n \simeq \left(\frac{m^2 M_p^2}{v^4} \right) \prod_n \left[\lambda_n \left(\frac{v^4}{m^2 M_p^2} \right) \right]^{V_n} , \tag{2.28}$$

because at tree level only $d_n = 0$ vertices can contribute to an effective interaction having $\tilde{d}_n = 0$. Expanding this tree-level result in graphs involving one, two and more vertices then gives[6]

$$\tilde{\lambda}_n \simeq \lambda_n + \left(\frac{v^4}{m^2 M_p^2} \right) \sum_{\text{graphs}} k_{mn} \lambda_n \lambda_m + \cdots , \tag{2.29}$$

where k_{mn} are calculable coefficients and the sum is over graphs for which $d_n = d_m = 0$ and $N_n + N_m = \tilde{N}_n + 2$. The ellipses indicate tree level graphs involving three or more vertices. Similarly, one-loop graphs involving only one vertex contribute (for $\tilde{d}_n = 0$),

$$\delta \tilde{\lambda}_n \simeq \frac{1}{(4\pi)^2} \sum_n \left\{ r_n c_n \left(\frac{m^4}{v^4} \right) + \frac{m^2}{M_p^2} \left[s_n \lambda_n + t_n g_n \left(\frac{m^4}{v^4} \right) \left(\frac{m}{M} \right)^{d_n - 4} \right] \right\} , \tag{2.30}$$

where r_n, s_n and t_n are calculable, and so on.

Notice that if $m \simeq v^2 / M_p$ then $m / M_p \simeq (v/M_p)^2 \ll m/v \simeq v/M_p \ll 1$, and $m^2 M_p^2 \simeq v^4$. This implies no suppression by scales between the terms in eq. (2.29), while in eq. (2.30) it makes the sums involving c_n and λ_n of the same order as one another, but larger than those involving g_n. On the other hand, if m is dialled up to $m \simeq v$, such as by taking $\lambda_n \simeq (M_p/v)^{N_n}$ for some vertices in the scalar potential, then the factor in eq. (2.29) becomes $(v^4/m^2 M_p^2)(M_p/v)^{N_m + N_n - \tilde{N}_n} = (v^2/M_p^2)(M_p/v)^{N_m + N_n - \tilde{N}_n} = \mathcal{O}(1)$. In the loop expression, however, it is the λ_n term that dominates (unsuppressed by powers of v/M_p) for corrections to the (M_p/v)-enhanced couplings, while the c_n and λ_n terms compete (again unsuppressed by v/M_p) for the corrections to the generic λ_n's. In all cases the g_n coupling is subdominant.

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[6]Notice that one-particle reducible graphs are allowed to contribute to the low-energy effective action, which is only required to be irreducible with respect to the cutting of *light* particle lines.

3 Applications to inflation

In applications to slow-roll inflation the background fields are time-dependent, and so among the important scales in the problem are the characteristic times over which the various fields vary appreciably. For the metric this is given by the Hubble scale

$$H = \frac{\dot{a}}{a} \simeq \frac{\sqrt{V}}{M_p} \simeq \frac{v^2}{M_p}, \tag{3.1}$$

while the evolution of the inflationary scalar is similarly characterized by the scale

$$\mu_\phi = \frac{\dot{\phi}}{\phi}. \tag{3.2}$$

During slow-roll inflation the scales μ_ϕ and H are related to one another by the slow-roll conditions, which state that the inflaton time derivative satisfies

$$\dot{\phi} \simeq \frac{V'}{H} \simeq \frac{M_p V'}{\sqrt{V}} \simeq \sqrt{\epsilon V} \simeq \sqrt{\epsilon}\, v^2. \tag{3.3}$$

Here

$$\epsilon = \frac{1}{2} \left(\frac{M_p V'}{V} \right)^2 \quad \text{and} \quad \eta = \frac{M_p^2 V''}{V}, \tag{3.4}$$

are the two slow-roll parameters [10], where the derivatives are taken with respect to the canonically normalized fields. They arise because a necessary condition for a slow roll is that both must be small: $\epsilon, |\eta| \ll 1$. Eq. (3.3) implies that during a slow roll the relative size of H and μ_ϕ, depends on the size of ϕ, with

$$\mu_\phi = \frac{\dot{\phi}}{\phi} \simeq \frac{\sqrt{\epsilon}\, v^2}{M_p} \simeq \sqrt{\epsilon}\, H \quad \text{if } \phi \simeq M_p, \qquad \mu_\phi = \frac{\dot{\phi}}{\phi} \simeq \sqrt{\epsilon}\, v \quad \text{if } \phi \simeq v. \tag{3.5}$$

The observation that the inflaton-gravity action is a part of the more general effective lagrangian, eq. (2.1), imposes often unspoken conditions on the domain of validity of any analysis that bases inflation on its classical solutions. It requires in particular that the inflationary motion must be adiabatic, which puts an upper limit on the inflationary time-scales: $\mu_\phi, H \ll M$. Indeed, regarding the effective theory as a derivative expansion breaks down if $H, \mu_\phi \simeq M$, because then terms involving powers of $R/M^2 \simeq (H/M)^2$ or $(\partial\theta)^2/M^2 \simeq (\mu_\phi/M)^2$ are not small.

For many inflationary models there is an important constraint that restricts the freedom to choose H and μ_ϕ arbitrarily. This constraint arises when primordial fluctuations are regarded as arising as quantum fluctuations of the inflaton during inflation. Agreement with the observed temperature fluctuations in the CMB requires the amplitude of curvature perturbations to have a specific amplitude $\Delta_{\mathcal{R}}^2|_{k^\star} = 2.445 \pm 0.096 \times 10^{-9}$ [2], where $k^\star = 0.002\,\mathrm{Mpc}^{-1}$.

When these perturbations are generated by quantum fluctuations in ϕ, then the quantity that controls their amplitude is $\delta = H^2/\dot{\phi} = (24\,\pi^2 \Delta_{\mathcal{R}}^2|_{k^\star})^{1/2}$, and so using the above estimates for $\dot{\phi}$ and H gives

$$\delta \simeq \frac{1}{\sqrt{\epsilon}} \left(\frac{v}{M_p} \right)^2 \simeq 7 \times 10^{-4}. \tag{3.6}$$

This provides the important relationship $v/M_p \simeq 0.03\,\epsilon^{1/4}$.

3.1 Corrections to inflationary scenaria

Eq. (2.14) allows an estimate of how the various effective interactions contribute to an inflationary scenario, provided E is chosen to be the largest scale in the problem.

Classical effects from higher effective interactions. The first modification to consider is the contribution of the various effective interactions in eq. (2.1) to the classical equations of motion. In the language of the estimate (2.14) this amounts to asking the relative size of various contributions in the classical limit (i.e. when $L = 0$). Eq. (2.14) shows that (provided $g_n \lesssim \mathcal{O}(1)$) higher-derivative interactions with $d_n \geq 4$ are suppressed by at least two powers of E/M_p, plus additional powers of E/M if $d_n > 4$. On the other hand, interactions with $d_n = 2$ are not particularly suppressed, and generically neither are interactions from the scalar potential. These two quantities must therefore be included exactly into the classical calculation. In particular, it is often a bad approximation to work in the small-field limit that is implicit when expanding the potential in powers of ϕ, and neglecting terms beyond a particular power (like quartic) when in the inflationary regime, as has recently been re-emphasized within the context of string theory [11].

Quantum contributions. A second question asks about the size of quantum corrections to the classical approximation. The size of these effects depends crucially on how massive are the particles whose quantum fluctuations are under study. In all cases eq. (2.14) applies (or (2.20) if the natural scale for ϕ is $\phi \simeq v$ rather than $\phi \simeq M_p$), with $E \simeq m$ for quantum fluctuations from particles whose mass satisfies $m \gg \mu_\phi, H$, while $E \simeq \max(\mu_\phi, H)$ for the quantum effects of particles satisfying $m \ll \mu_\phi, H$.

Heavy particles: the limit $E \simeq m \gg H \simeq v^2/M_p$ leads to the estimates of section 2.3, with the additional information that $v^4/(E^2 M_p^2) \simeq v^4/(m^2 M_p^2) \simeq H^2/m^2 \ll 1$. This shows that in addition to the generic loop factor $(m/4\pi M_p)^2$, the $d_n \geq 4$ interactions — g_n — are further suppressed by at least two powers of m^2/M_p^2, and interactions in the scalar potential — λ_n — are additionally suppressed by powers of H^2/m^2. Only the $d_n = 2$ interactions — c_n — remain unsuppressed beyond the basic loop factor if $\lambda_n \lesssim \mathcal{O}(1)$. On the other hand, if there are interactions in the scalar potential that are unsuppressed by powers of M_p (such as if $\lambda_n \simeq (M_p/v)^{N_n} \check{\lambda}_n$, as discussed above) then loops involving these interactions can also modify the inflaton mass in a dangerous way [13].

Provided the heavy field itself only moves adiabatically, the implications of loop effects of this type are most simply seen by integrating the particle out, leading again to an effective theory of the form of eq. (2.1), but with M replaced by m [12–15]. As we have seen, only those interactions having two or fewer derivatives generically have an appreciable influence on the classical equations, since the effects of interactions with $d_n \geq 4$ have been argued already to be small. In general, quantum corrections can change the shape of the classical potential, and such changes can ruin the inflationary slow roll of the original potential unless they are absorbed into the coefficients of the coefficients of the original effective action. This is particularly true when ϕ arises in the scalar potential V suppressed by a light scale like v rather than M_p. This simply represents the usual naturalness problems in keeping low-dimension terms in the scalar potential small as heavier particles are integrated out.[7]

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[7]Approximate symmetries, such as shift symmetries [16], can protect the size of such corrections, al-

Unfortunately, although these corrections need not be small, and can undermine whether or not we believe a given theory actually exhibits inflation in the first place, they do not have *observable* implications in the sense that cosmological observations are unable to separate quantum from classical contributions to the potential. On the other hand, if the heavy-field motion is not adiabatic, it need not decouple and so cannot be integrated out. In this case its presence can generate observable deviations from standard inflationary predictions [17].

Light particles: the analysis is different when the mass of the particle in the loop is small compared with H and μ_ϕ, which includes in particular the inflaton itself since its mass is $m^2 = V'' = \eta V/M_p^2 \simeq \eta v^4/M_p^2 \simeq \eta H^2 \ll H^2$. In this case the estimate (2.14) still applies, but it is $E \simeq H$ (since $\mu_\phi \simeq \sqrt{\epsilon}\, H \ll H$ in this case) that should be used.

Specializing eq. (2.14) to $E \simeq H$ then gives

$$
\mathcal{A}_{\mathcal{E}}(E) \simeq H^2 M_p^2 \left(\frac{1}{M_p} \right)^{\mathcal{E}} \left(\frac{H}{4\pi\, M_p} \right)^{2L} \prod_{d_n=2} \left(c_n \right)^{V_n} \tag{3.7}
$$
$$
\times \prod_{d_n=0} \left(\lambda_n \right)^{V_n} \prod_{d_n \geq 4} \left[g_n \left(\frac{H}{M_p} \right)^2 \left(\frac{H}{M} \right)^{d_n-4} \right]^{V_n} ,
$$

where $\lambda_n \lesssim \mathcal{O}(1)$ provided V/v^4 varies appreciably only when ϕ changes by an amount of order M_p. This shows the irrelevance of the g_n terms (having 4 or more derivatives), as well as the lack of additional suppression of the $d_n = 2$ and $d_n = 0$ interactions, beyond the basic loop-suppression factor.

To apply this to the one-loop inflaton fluctuations themselves, $\langle \phi^2 \rangle$, recall that the quartic interaction in the scalar potential is $\lambda_4 v^4 (\phi/M_p)^4 \simeq \lambda_4 (H/M_p)^2 \phi^4$. The one-loop graph involving this vertex contributes an amount of order $\lambda_4 (H/M_p)^2 \langle \phi^2 \rangle$ to the 2-point function, which can be compared with eq. (3.7) specialized to $\mathcal{E} = 2$ to read off the size of $\langle \phi^2 \rangle$. This gives the estimate

$$
\langle \phi^2 \rangle \simeq \left(\frac{H}{4\pi} \right)^2 , \tag{3.8}
$$

in agreement with the standard calculations. Indeed it is this connection between $\langle \phi^2 \rangle$ and H^2 that is responsible for the numerator of the observable combination $\delta = H^2/\dot{\phi}$ discussed earlier. (The $\dot{\phi}$ comes from the requirement that the ϕ fluctuation mix with the metric to generate a curvature fluctuation that can be observed in the CMB.)

3.2 Applications

As an example of the utility of these power-counting estimates we apply the above reasoning to identify the domain of validity of semiclassical methods in two closely related inflationary models.

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though it is important that these symmetries apply to *all* couplings of the inflaton and not just to the self-couplings that appear in the inflaton potential.

3.2.1 Higgs inflation

Using this formalism, we now consider the example of Higgs inflation [3] that has recently gained some attention [18, 19]. This model starts with the very economical proposal to try to obtain inflation using the Standard Model Higgs as the inflaton. The idea is to do so by supplementing the Standard Model and Einstein-Hilbert lagrangian densities with the sole dimension-4 interaction that is not normally written down:[8]

$$\mathcal{L}_{H\,\mathrm{inf}} = \mathcal{L}_{SM} + \mathcal{L}_{EH} + \xi\,\mathcal{H}^\dagger\mathcal{H}\,R\,, \tag{3.9}$$

where $\mathcal{H}$ is the usual Standard Model doublet $\sqrt{2}\,\mathcal{H} = (0, v_H + h)^T$, and ξ is a dimensionless coupling. In particular, the Higgs potential is the usual quartic form,

$$V = \lambda_H\left(\mathcal{H}^\dagger\mathcal{H} - \frac{v_H^2}{2}\right)^2 = \frac{\lambda_H}{4}\left(2\,v_H h + h^2\right)^2\,, \tag{3.10}$$

where λ_H is related to the Higgs boson mass by $m_H^2 \simeq 2\lambda_H v_H^2$. Because the rest of the action is completely determined by non-inflationary physics, the only adjustable parameter with which to try to make the model inflate is ξ.

Once one performs a Weyl rescaling to transform to the Einstein frame the Higgs potential becomes

$$V_{EF} \simeq \frac{\lambda_H(\mathcal{H}^\dagger\mathcal{H} - v_H^2/2)^2}{(1 + \xi\,\mathcal{H}^\dagger\mathcal{H}/M_p^2)^2}\,, \tag{3.11}$$

which is to be regarded as being a function of $h(\phi)$, where ϕ is the field that canonically normalizes the Einstein-frame Higgs kinetic term. Remarkably, this can be flat enough to inflate, provided that there is a reliable regime for which $\mathcal{H}^\dagger\mathcal{H} \gg v_H^2$ and $\xi\mathcal{H}^\dagger\mathcal{H} \gg M_p^2$, since in this case $V_{EF} \simeq \lambda_H M_p^4/\xi^2$ is approximately constant.

More precisely, expressing the potential in terms of the canonical variable in the inflationary regime gives

$$V_{EF} \simeq \frac{\lambda_H M_p^4}{\xi^2}\left[1 + Ae^{-a\phi/M_p}\right]^{-2}\,, \tag{3.12}$$

where A and a are dimensionless numbers. The inflationary regime of interest is then $\phi \gg M_p$, since in this case $[1 + Ae^{-ax}]^{-2} \simeq 1 - 2Ae^{-ax} + \cdots$ is approximately constant. Dropping $\mathcal{O}(1)$ constants, this shows that the energy density during inflation is $V \simeq v^4$ where $v^2 \simeq \sqrt{\lambda_H}M_p^2/\xi$, and so the Hubble scale during inflation is $H \simeq v^2/M_p \simeq \sqrt{\lambda_H}M_p/\xi$. Computing the value of the slow-roll parameters at horizon exit and demanding $\delta \simeq 7\times10^{-4}$ then shows that the amplitude of primordial fluctuations agrees with observations provided

$$\xi \simeq 5\times10^4\,\sqrt{\lambda_H} \simeq 5\times10^4\left(\frac{m_H}{\sqrt{2}\,v_H}\right) \gg 1\,, \tag{3.13}$$

where $m_H > 115\,\mathrm{GeV}$ and $v_H = 246\,\mathrm{GeV}$ respectively denote the mass and expectation value of the Higgs [3].

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[8]Earlier examinations of non-minimally coupled models include [20].

The large size of the coupling ξ is unusual from the particle physics perspective, and leads one to worry about whether multiple insertions of the corresponding effective interactions might generate unexpectedly large quantum effects. This is the kind of question for which the above power-counting arguments are well suited. Although the size of some loop effects were examined in refs. [18, 19], the generality of the power-counting result given earlier allows the effects of couplings to be identified systematically.

Since the Einstein-frame potential energy varies by of order v^4 when ϕ ranges through the range M_p the power-counting result, eq. (2.14), may be directly applied. In particular, it can be used to put an upper bound on the energy scale, M, at which the low-energy effective description must break down. This is most easily done by studying energetic graviton-Higgs scattering, $gh \rightarrow gh$, or Higgs-Higgs scattering, $hh \rightarrow hh$, in flat space, and asking when this saturates the unitarity bound as a function of the loop order L. For this purpose we may apply eq. (2.14) to the scattering amplitude, taking E to be the center-of-mass energy of the scattering. Furthermore, because we expand about flat space and small Higgs vev we may regard the interaction $\xi \mathcal{H}^\dagger \mathcal{H} R$ as an interaction vertex involving $d_n = 2$ derivatives.

To obtain the bound we concentrate on the potentially most dangerous graphs that involve only the coupling ξ. According to eq. (2.14), an L-loop graph of this type that involves V_n insertions of the ξ coupling constant contributes to the ($\mathcal{E} = 4$)-point amplitude an amount

$$\mathcal{A}_4(E) \simeq \left(\frac{E}{M_p}\right)^2 \left(\frac{E}{4\pi M_p}\right)^{2L} \prod_n \xi^{V_n}, \tag{3.14}$$

where the product is over the power, N_n, of the fields h and $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ appearing in the expansion of the original interaction $\xi \mathcal{H}^\dagger \mathcal{H} R$.

By virtue of the identity, eq. (2.19), the quantities V_n and N_n are related to L and $\mathcal{E} = 4$ by $\sum_n (N_n - 2) V_n = \mathcal{E} - 2 + 2L = 2 + 2L$, and so the largest power of ξ at any fixed loop order arises from multiple insertions of the $N_n = 3$ vertex, in which case $V_{\max} = 2 + 2L$. The highest power of ξ appearing at any fixed order in L then becomes

$$\mathcal{A}_4^{\max}(E) \simeq \left(\frac{\xi E}{M_p}\right)^2 \left(\frac{\xi E}{4\pi M_p}\right)^{2L}. \tag{3.15}$$

At tree level this gives $\mathcal{A}_{4,tree} \propto \xi^2$, corresponding to the scattering graph involving two trilinear $h - h - h_{\mu\nu}$ vertices. (Notice that for graviton-Higgs scattering this is a stronger dependence than the linear dependence in ξ coming from the naive graph involving no internal lines at all, that uses the quartic $h - h - h_{\mu\nu} - h_{\lambda\rho}$ vertex, demonstrating the utility of the power counting analysis.)

Demanding that the cross section built from a term like this not saturate the unitarity bound, $\sigma \propto 1/E^2$, gives a ξ-dependent upper bound on how large E can sensibly be within the low-energy theory, leading to

$$E < E_{\max} \simeq \frac{M_p}{\xi}. \tag{3.16}$$

For Higgs-Higgs scattering through graviton exchange this power-counting estimate reproduces the results of an explicit calculation [21], with the $\mathcal{O}(1)$ numerical factor not written

explicitly in eq. (3.16) revealed to be $\sqrt{\pi/6}$. This provides a quantitative upper bound on the true cut off of the theory.

Eq. (3.16) is useful because it furnishes an upper bound as to how big the scale M can be that controls the size of higher-derivative terms in the low-energy effective theory.[9] Some new physics must intervene at a scale $M < M_p/\xi$, so long as the more microscopic underlying physics whose low-energy sector the effective theory captures is itself unitary. Because the Hubble scale is $H \simeq \sqrt{\lambda_H} M_p/\xi$ in this picture, the identification $M \lesssim M_p/\xi$ implies $H/M \gtrsim \sqrt{\lambda_H}$. This leaves only the narrow window $1 \gg H/M \gg \sqrt{\lambda_H}$ within which all approximations remain valid.[10]

This window gets more uncomfortable the more the new physics couples to the Higgs field, since we've seen that the approximately constant inflationary potential relies on there being a regime for which $V \propto (\mathcal{H}^\dagger \mathcal{H})^2$ and the non-minimal coupling to gravity is fR with $f \propto \mathcal{H}^\dagger \mathcal{H}$. Although this is the case for quartic V and quadratic f when $\mathcal{H}^\dagger \mathcal{H} \gg M_p^2/\xi \gg v_H^2$, it need no longer remain so once terms of order $\delta V \propto (\mathcal{H}^\dagger \mathcal{H})^3$ or $\delta f \propto (\mathcal{H}^\dagger \mathcal{H})^2$ (or higher) are generated by loops. Furthermore, as is seen from eqs. (2.29) and (2.30), these corrections generically need not be small.

For instance, a quartic coupling of the form $g\mathcal{H}^\dagger \mathcal{H} \chi^\dagger \chi$ between the Higgs and a heavy field χ having mass M_χ cannot be forbidden by any internal symmetries and would generate loop contributions $\delta V \simeq g^3 (\mathcal{H}^\dagger \mathcal{H})^3/(4\pi M_\chi)^2$ and $\delta f \simeq g^2 (\mathcal{H}^\dagger \mathcal{H})^2/(4\pi M_\chi)^2$. The quartic term in V can only dominate if $g^3 \mathcal{H}^\dagger \mathcal{H}/(4\pi M_\chi)^2 \ll \lambda_H$ and similarly the quadratic term in f dominates if $g^2 \mathcal{H}^\dagger \mathcal{H}/(4\pi M_\chi^2) \ll \xi$. Using $\mathcal{H}^\dagger \mathcal{H} \gg M_p^2/\xi$ in these conditions shows that the scale, Λ, suppressing higher powers of the Higgs field must satisfy $\Lambda \simeq 4\pi M_\chi/g \gg M_p \sqrt{g/(\lambda_H \xi)}$ (for V) and $\Lambda \gg M_p/\xi$ (for f). For $g \simeq \lambda_H$ the first of these shows — not surprisingly — that Λ must be greater than the typical size of the Higgs field during inflation, $\Lambda \gg M_p/\sqrt{\xi}$. The second shows that this bound does not get worse than $\Lambda \gg M_p/\xi$, even if g/λ_H should be smaller than $1/\xi$. That is,

$$\Lambda \gg M_p \sqrt{\frac{g}{\lambda_H \xi}} \quad \text{(if } g > \lambda_H/\xi) \qquad \text{or} \qquad \Lambda \gg \frac{M_p}{\xi} \quad \text{(if } g < \lambda_H/\xi). \tag{3.17}$$

On the other hand, within this model it is the same mass scale, M_χ, that ultimately suppresses generic higher-derivative terms in the effective action, since higher-curvature terms are also generated at one loop of the form $R^3/(4\pi M_\chi)^2$. We see that the quantity M in the effective theory obtained by integrating out χ is of order $M \simeq 4\pi M_\chi$, and so unitarity requires $4\pi M_\chi \ll M_p/\xi$, or

$$\Lambda \ll \frac{M_p}{g\xi}. \tag{3.18}$$

Consider now the separate cases $g < \lambda_H/\xi$ and $g > \lambda_H/\xi$. If $g > \lambda_H/\xi$ then conditions (3.17) and (3.18) together require $1/(g\xi) \gg \sqrt{g/(\lambda_H \xi)}$, or $g^3 \ll \lambda_H/\xi < g$. On the other

hand, if $g < \lambda_H/\xi$ then (3.17) and (3.18) together simply require $g \ll 1$. We see explicitly in this example how any other particles must be kept very heavy and/or strongly sequestered from the Higgs in order for the inflationary mechanism to be viable.[11]

3.2.2 Inflation from curvature-squared terms

As our second application we next examine inflationary proposals that are based on higher-curvature interactions [22], which represent a variation on the above theme. Consider to this end the curvature-squared action

$$\mathcal{L} = \sqrt{-g} \left[-\frac{M_p^2}{2} R + \zeta R^2 \right]. \tag{3.19}$$

The Hubble scale can be most easily identified by exploiting the relationship between this theory and the Higgs-Inflation theory. This can be made clear by rewriting the R^2 Lagrangian as a scalar-tensor model by performing a Hubbard-Stratonovich transformation and 'integrating in' a scalar field of dimension one Φ, as in

$$\mathcal{L} = \sqrt{-g} \left[-\frac{M_p^2}{2} R - 2\,\alpha\,\Phi^2\,R - \Phi^4 \right]. \tag{3.20}$$

Performing the gaussian integral over Φ returns the lagrangian density of eq. (3.19), with $\zeta = \alpha^2$.

The relation between this model and the one previously considered can be seen by performing a conformal transformation on this theory to the Einstein frame $g_{\mu\nu}^E = f(\Phi)g_{\mu\nu}$ with $f(\Phi) = 1 + 4\alpha\,\Phi^2/M_p^2$ such that the Lagrangian becomes,

$$\mathcal{L} = \sqrt{-g_E} \Big(-\frac{1}{2}M_p^2\,R_E - \frac{3}{4}M_p^2\,\frac{f'(\Phi)^2}{f(\Phi)^2}\,(\partial_E\Phi)^2 - V_E(\Phi) \Big) \tag{3.21}$$

where the Einstein-frame scalar potential is

$$V_E(\Phi) = \frac{\Phi^4}{\left(1 + \frac{4\alpha}{M_p^2}\Phi^2\right)^2}. \tag{3.22}$$

Further transforming to a canonical scalar field σ through the field transformation

$$\sigma = \sqrt{\frac{3}{2}}M_P \ln\left(1 + 4\alpha\,\Phi^2/M_p^2\right), \tag{3.23}$$

the Einstein-frame scalar potential becomes

$$V_E(\sigma) = \frac{M_P^4}{16\alpha^2} \left[1 - \exp\left(-\sqrt{\frac{2}{3}}\sigma/M_P\right) \right]^2. \tag{3.24}$$

---

[11]The potential danger of these interactions, and the potential necessity for there to be a desert involving no such virtual particles up to these large scales was already recognized in the original literature.

The inflationary analysis therefore proceeds much as in Higgs inflation before, with inflation occurring for fields $\Phi \gg M_p/2\sqrt{|\alpha|}$ or $\sigma \gg \sqrt{\frac{3}{2}}M_P$, where the Einstein-frame scalar potential is of order $V_{EF} \simeq \lambda M_p^4/(16\xi^2) = M_p^4/16\zeta$ and the Hubble scale is $H \simeq \sqrt{\lambda}\, M_p/\xi \simeq M_p/4\sqrt{\zeta}$. Again, successful generation of primordial density fluctuations requires the combination $\xi/\sqrt{\lambda} = 4\,|\alpha| = 4\,\sqrt{\zeta}$ to be large, of order 10^4.

To see when $\zeta \simeq 10^8$ begins interfering with the semiclassical approximation we again use the power-counting arguments of previous sections. There are two equivalent ways to determine the bounds on E for this theory, one can directly analyze the given lagrangian and calculate the cut off scale for graviton-graviton scattering, $gg \to gg$, using the $d_n = 4$ interactions of the lagrangian density. Repeating the arguments used for Higgs Inflation above leads to a problem with unitarity once the scattering energies reach $E \simeq E_{\max} = M_p/\zeta^{1/3}$. Alternatively, one can use the theory after the Hubbard-Stratonovich transformation in the einstein frame and power-count. Note that one wishes to power count interactions with no external Φ fields as in this case Φ is an auxiliary field and not a real field as in Higgsflation. One can construct effective $d_n = 4$ interaction operators and then power count directly as before, again obtaining a cut off scale $E \simeq E_{\max} = M_p/\zeta^{1/3}$.

In either approach, we require that the scale M controlling all other powers of curvature not written explicitly in eq. (3.19) to satisfy $M \ll M_p/\zeta^{1/3}$. But using the above expression for the inflationary Hubble scale, $H \simeq M_p/\sqrt{\zeta}$ then shows that the ratio H/M must satisfy $H/M \gg \zeta^{-1/6} \simeq 1/20$. Again inflation requires H/M to be close to a breakdown of the adiabatic approximation that underlies the understanding of eq. (3.19) as part of a low-energy effective theory, making any inflationary conclusions drawn using it somewhat suspect.

4   Conclusions

Quantum corrections to the semi-classical approximation generally employed of the inflation literature can be critical in determining the viability of particular inflationary scenarios. Indeed, it is the fact that quantum effects are *not* completely negligible that underlies the possibility of explaining primordial fluctuations in terms of quantum fluctuations of the inflaton.

Although calculating quantum effects in non-renormalizable theories like gravity may be unfamiliar, there is a well-defined framework within which it may be done. This framework was developed and tested against experiment using non-renormalizable theories elsewhere in physics, and relies on the observation that the semiclassical limit in such cases is controlled by a low-energy approximation.

In this paper we have applied standard power-counting arguments for such theories that allow one to easily quantify the domain of validity of the classical approximation within any particular model. Indeed, it is because many slow-roll models lie well within the classical limit that justifies the belief that inflation can reliably be predicted using the standard classical analyses.

However the same may not be true for more exotic inflationary models, or for models of dark energy for that matter, almost all of which are founded on a purely classical analysis. We believe it behooves the proponent of any such a scenario to justify that validity of the classical approximation, which should be viewed as one of the hurdles any serious proposal must clear.

As an application of these techniques, we have examined the domain of validity of the Higgs-Inflaton scenario, and find that its semiclassical analysis is consistent only if the scale, M, governing the low-energy approximation lies in the narrow range, $M_p/\xi \gg M \gg \sqrt{\lambda_H} M_p/\xi$, where ξ is the coefficient of the non-minimal Higgs-graviton interaction, $\xi \mathcal{H}^\dagger \mathcal{H} R$, and $\lambda_H = m_H^2/(2 v_H^2)$ is the usual Standard Model Higgs quartic self-coupling. Although it is a logical possibility that such a scale exists, we argue that it is extremely unstable to the existence of any small couplings between the Higgs and other heavy particles. The situation is similar for curvature-squared inflationary models, which also must push the adiabatic approximation that is essential to regarding such theories as well-behaved low-energy effective descriptions of any sensible underlying microscopic dynamics.

Acknowledgments

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